Decision Making Under Risk and Uncertainty

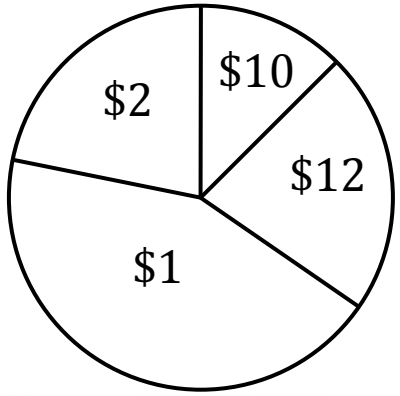
Sarabi-Jamab (IPM)

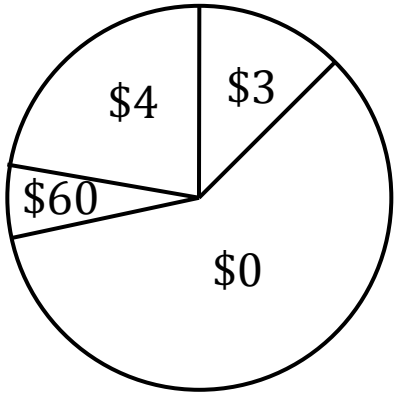
atiye.Sarabi@gmail.com

Roadmap

- **What is Decision Making ?**
- **Historical contributions to the uncertain-based models**
- **Neuroanatomical Substrates for Risk and Ambiguity Behavior**

Decision making

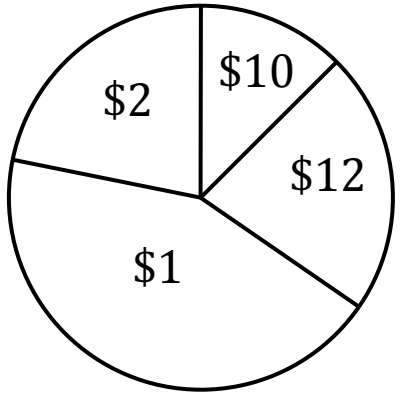


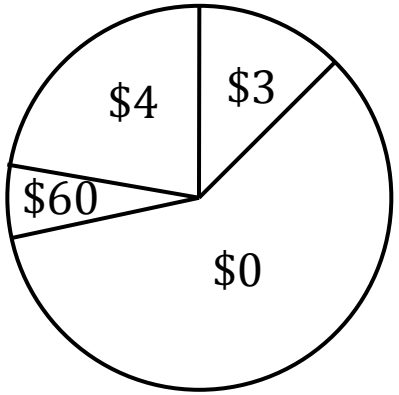


Choices are lotteries consisting of

- utilities
- probabilities

Decision making

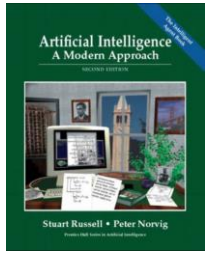




= probability theory + utility theory

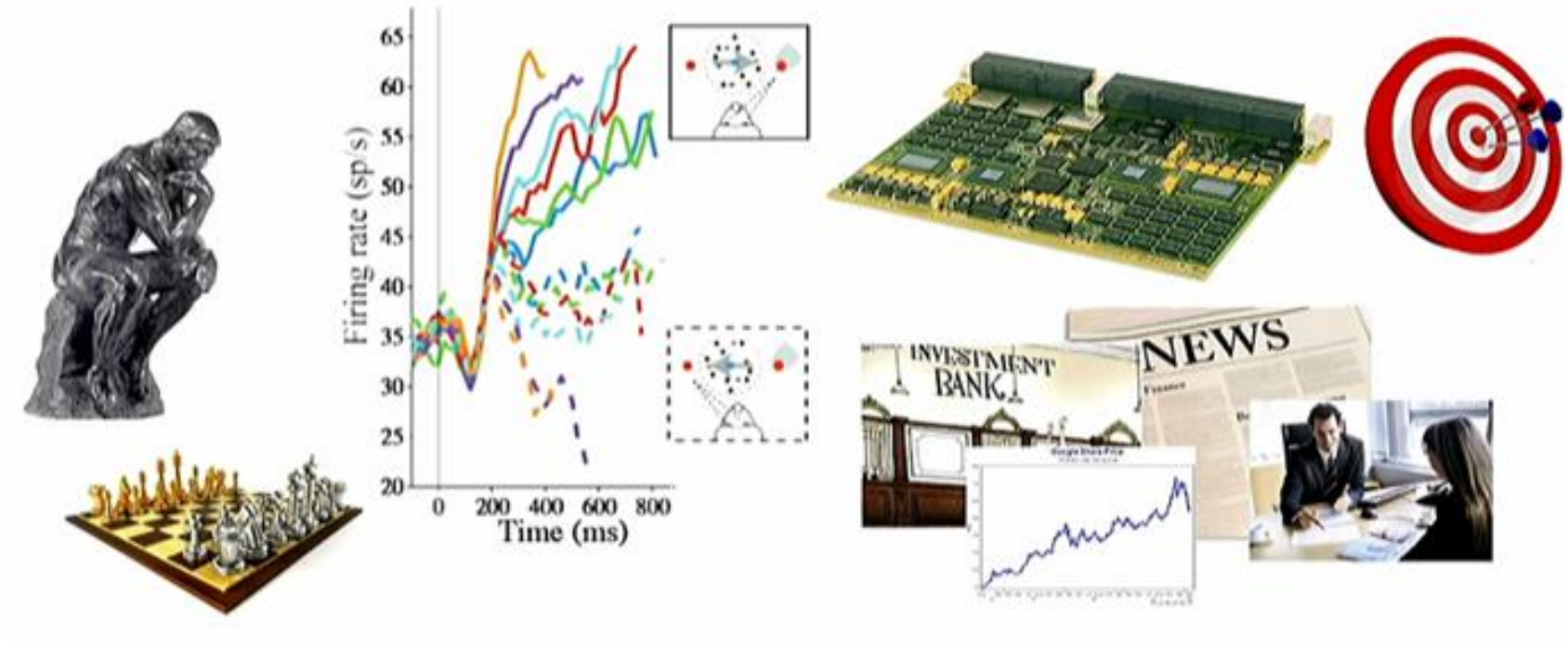
Choose lottery **i** with highest Expected Utility over Outcomes **s**

$$EU(i) = \sum_s P(s|i)U(s)$$



For each possible percept sequence a **rational agent** selects an action that is expected to maximize the performance measure given the percept sequence and prior knowledge

Real-world decision-making are limited



Real-world decision-makers can not simply, choose because they have limited information prosessing resources and model uncertainty

What is degree of uncertainty?

In the economic literature there is a longstanding debate about known vs. unknown uncertainty

risk **ambiguity**

? whether these two kinds of uncertainty are the same or whether they are processed in a different way by human decision-makers

Type of Uncertainty: Objective vs. Subjective

Objective or irreducible:
Inherent stochasticity in physical or biological system
Externally given probabilities, can be measured.
Classical example: roulette wheel

Subjective or reducible:
Probabilities specific to the decision maker, discoverable through revealed preferences.
Classic example: horse race

Decision making models

- **Normative** or perspective models (how decision should be made in order to be rational?)
  (Maximize expected value or …)

- **Descriptive** models (what people with different attitudes actually accomplish in case of decision-making?)
  (like prospect theory)

descriptive models of Decision-Making try to decision the *variability at different phenomenon of attitude* rather than providing an *optimum solution*

Historical contributions to DM models

1670	Pascal: Degree of belief
1737	Bernoulli: Utility
…	
1921	Knight: Uncertainty & Risk
1926	Ramsey: Willingness to bet Subjective probability
1931	De finetti: Qualitative probability
1938	Samuelson: Revealed Preference
1944	Morgenstern/Von Neuman: Objective Expected Utility
1954	Savage: Subjective Expected Utility
1961	Ellsberg: Ellsberg Paradox
1967	Dempster: Imprecise & non-additive probability
1989	Gilboa & Schmidler: non-additive probability

Decision making models

Pascal suggested that the desirability of any option that the decision maker considers is equal to the value of that option multiplied by the probability of obtaining that value:

Expected Value = Probability ×Value

By that account, to make a choice between several options, the decision maker needs simply to compute the expected value of each option and choose the option of the highest value.

✗ This simple concept is, of course, *too* simple

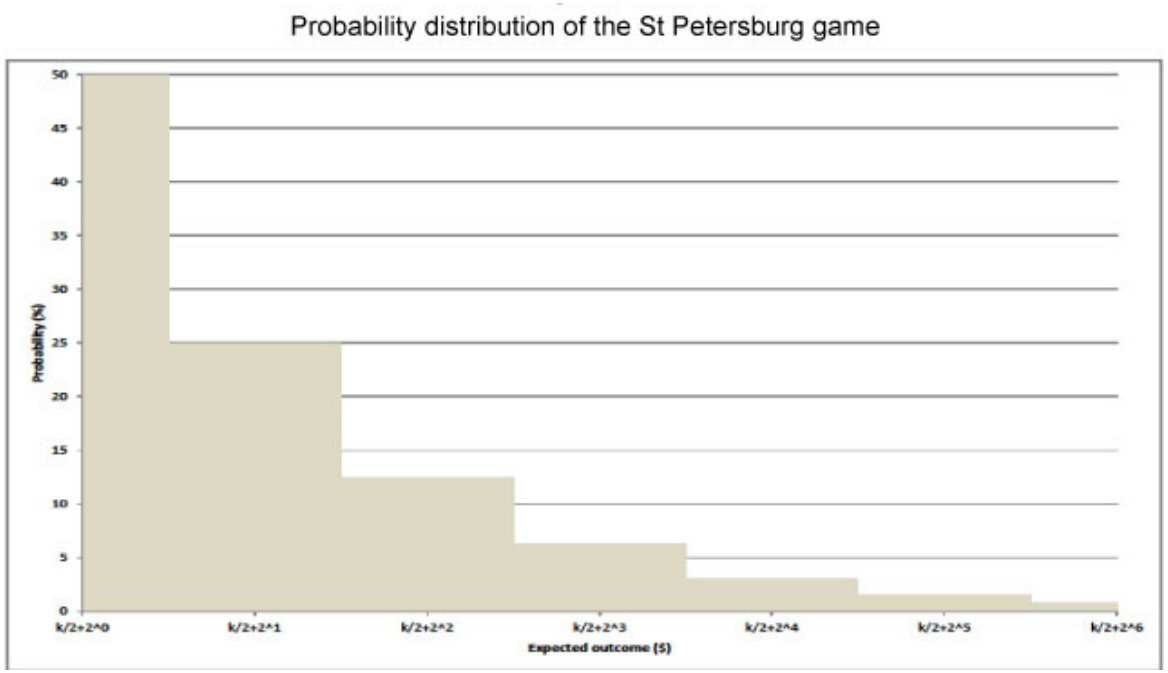
Blaise Pascal: Degree of belief (1670)

Decision making models

- Gambler's Wisdom: maximize expected (monetary) return
- St. Petersburg Paradox:
  - Toss fair coin until "head" shows up.
  - The payoff doubles each round, hence expected return is infinite:

$$\mathbb{E}[R] = \frac{1}{2} \cdot 1 + \frac{1}{4} \cdot 2 + \frac{1}{8} \cdot 4 \cdots = \infty$$

but people are founded to be willing to pay only small price for playing

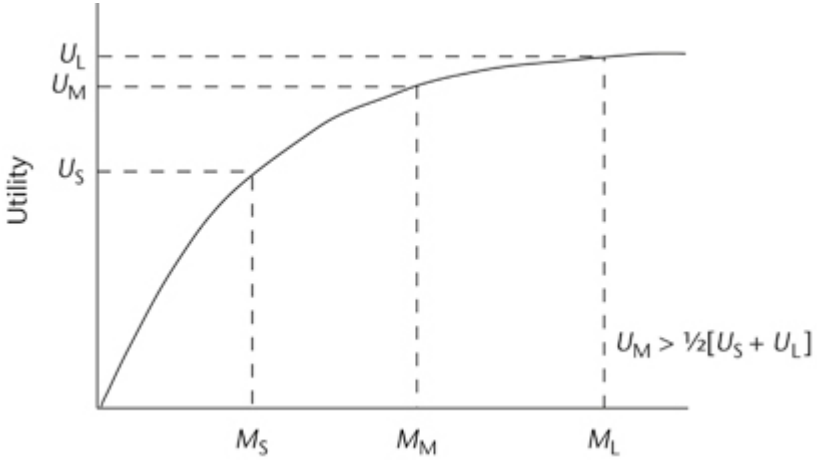


Decision making models

Bernoulli: the Expected Utility Hypothesis (1728)

- Daniel Bernoulli postulates hypothesis with two ideas”
  - Value is subjective: Utility
  - Utility is marginally diminishing (e.g. logarithmic in the return)

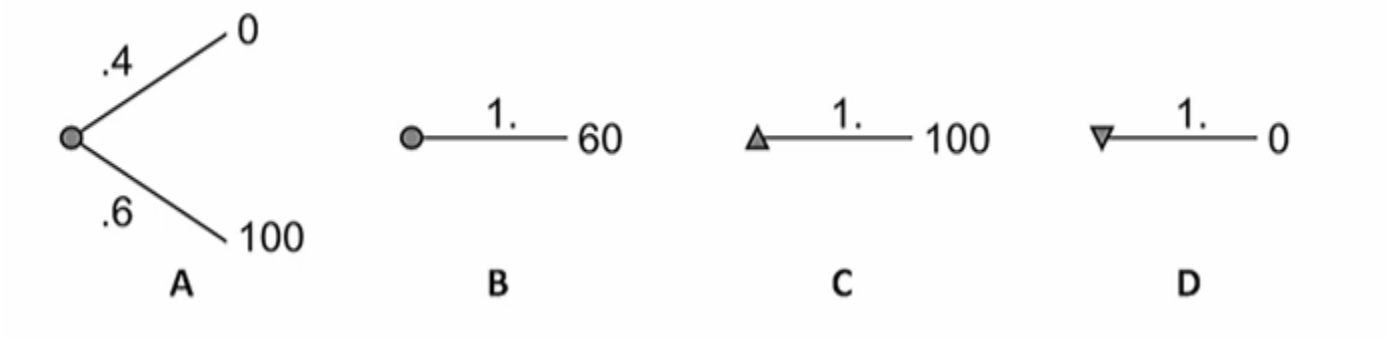
Bernoulli made this point using an example of a very poor fellow that obtains a lottery ticket with an equal probability to win either 20,000 ducats or nothing. Should this man evaluate his chance of winning at 10,000 ducats (its expected value), asks Bernoulli?
Or should he be willing to accept a smaller amount, say 9,000 ducats, in exchange for the lottery ticket? The intuitive answer, that this man should accept the 9,000 ducats, suggests that *expected value* is not a sufficient quantity for decision making.



Decision making models

Certainty- Equivalent (CE)

Guaranteed amount of utility that an individual sees as equally desirable to a given uncertain utility. In some approaches (e.g. RL) it is called Value.
Every choice has a CE:



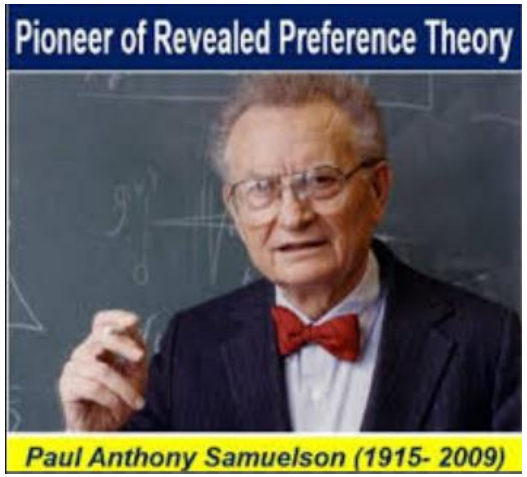
Note: Expected Utility Theories can be characterized in term of their CE: it is the expectation operator

Decision making models

Samuelson (1938)

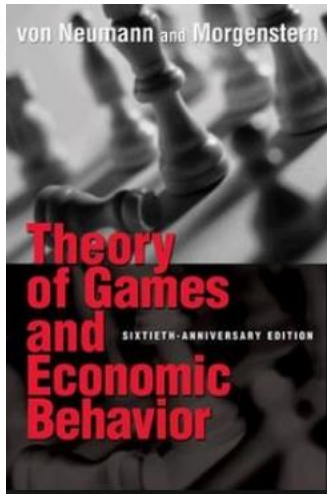
Samuelson has introduced the powerful idea of revealed preferences.

What Samuelson realized was that by making some very simple assumptions.
for example, that if a person prefers an apple to an orange she will not also prefer an orange to an apple, one can make robust predictions about choice behavior.



Decision making models

Von Neumann/ Morgenstern: Expected Utility Theory (1944)



- Axiomatization of utility based on revealed preferences
- Individuals have preferences over probability distributions over outcomes (can be anything, not just money)
- Probabilities are objective, Nature-given
- Representation theorem: Preference relation obeys rationality axioms which lead to subjective utility function

Rationality Axioms:

- **Completeness:**
  Either $L \succ M$, $M \succ L$, or $L \sim M$
- **Transitivity:**
  If $L \succcurlyeq M$ and $M \succcurlyeq N$ then $L \succcurlyeq N$
- **Continuity:**
  If $L \succcurlyeq M \succcurlyeq N$, then there exists $p \in [0, 1]$ such that $pL + (1\text{-}p)N \sim M$
- **Independence:**
  If $L \succcurlyeq M$, then for any L and $p \in [0, 1]$, $pL+(1\text{-}p)N \succcurlyeq pM + (1\text{-}p)N$.

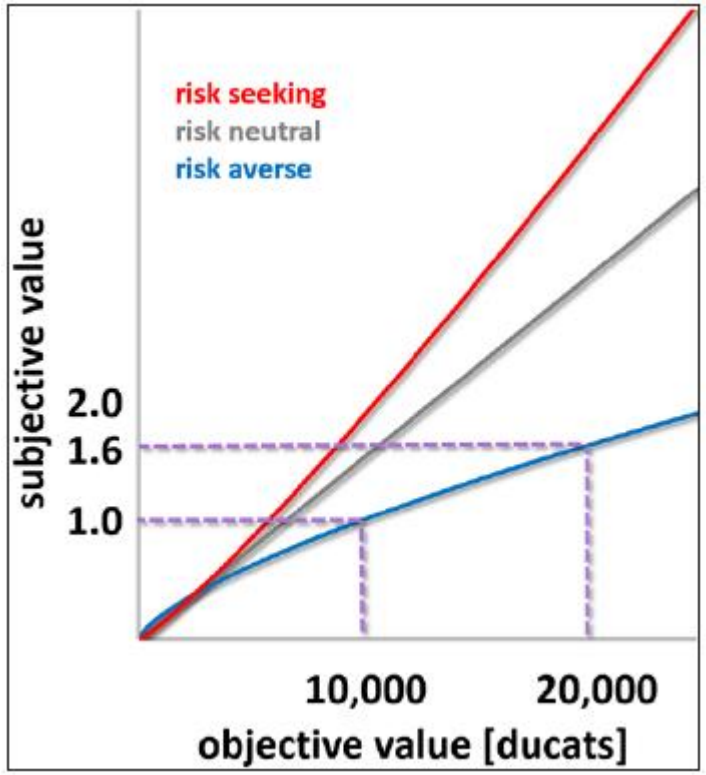
✗ axioms are often violated (such violation was provided by Daniel Ellsberg (1961)).

Decision making models

Rather, the *expected utility* or *subjective value* of an option should be considered. A subjective value function that is concave in respect to the objective value (the amount of ducats) can account for the risk preference we expect the poor fellow to exhibit. What happens in a concave value function is that subjective value increases more slowly than objective value. A power function of the following form:

$$Subjective\ Value = Probability \times Value^{a}$$

could account for individual differences in risk preferences with a single parameter (α). Risk-averse behavior will be described with an α that is smaller than 1, while larger than one will capture risk-seeking behavior

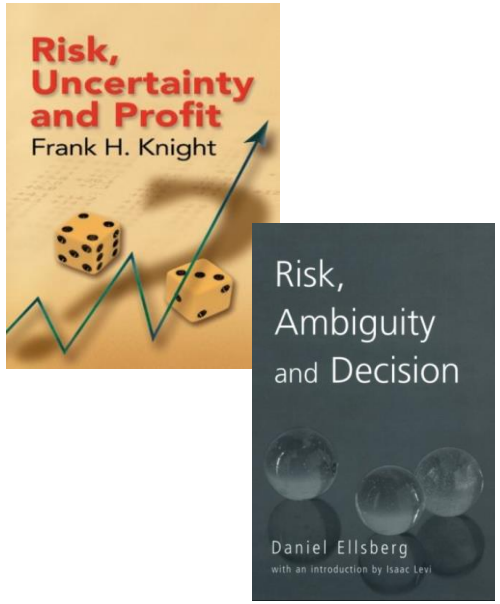


Decision making models

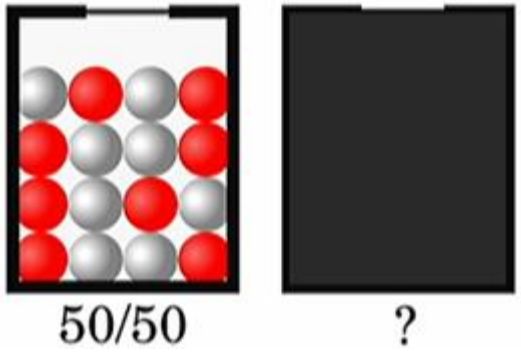
Knight/ Ellsberg: Unknown Uncertainty/ Ambiguity (1921/1961)

Knight distinguished between:

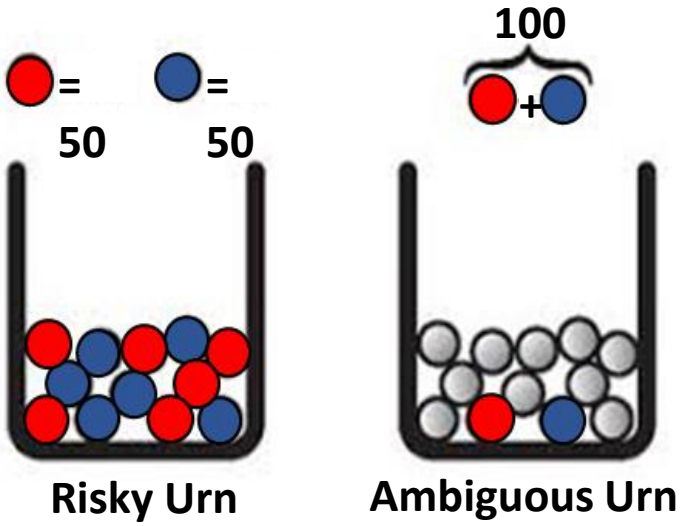
- Risk: Measurable uncertainty (both objective and subjective)
- “Knightian” Uncertainty: Non-measurable uncertainty.



Ellsberg Paradox: Subjects tend to prefer non-ambiguous urn independent of winning color



Ellsberg Paradox



Task 1:
Choose between two urn by Setting a prize on drawing a blue ball

Result
Players tend to prefer choosing the risky urn over the ambiguous urn

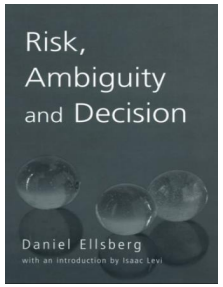
$\mathrm{N}(B)$ in A urn < 50

$N(B \cup R) < 100$

Task 2:
Choose between two urn by Setting a prize on drawing a red ball

Result
Players tend to prefer choosing the risky urn over the ambiguous urn

$\mathrm{N}(R)$ in A urn < 50



Decision making models

Economists distinguish between two types of theories:

- **Normative** or perspective models (how decision should be made in order to be rational?)
- **Descriptive** How individuals actually choose models (what people with different attitudes actually accomplish in case of decision-making?)

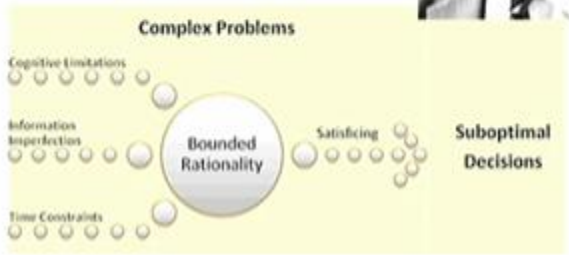
Gilboa & Schmidler: non-additive probability

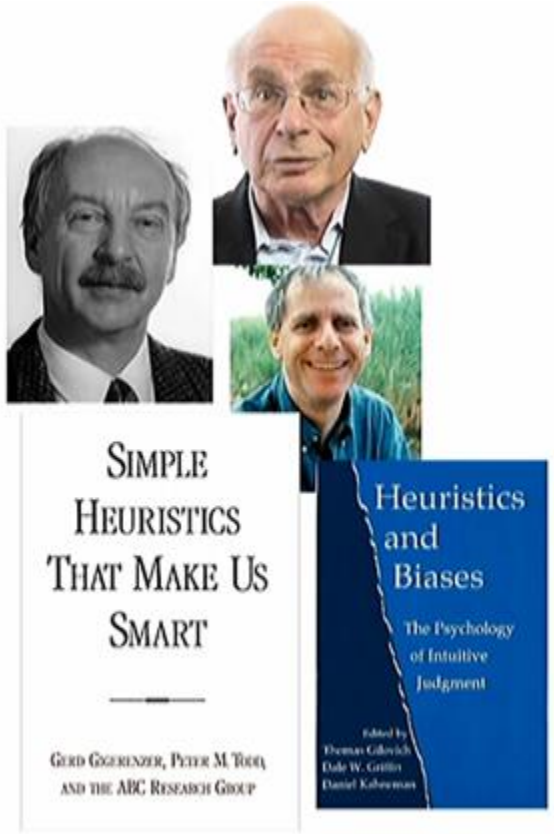
$$SubjectiveValue = [p - \beta(\frac{A}{2})] \times V^{\alpha}$$

Recap : Decision making models

Bounded rational choice theories

"[R]ationality is bounded when it falls short of omniscience. And the failures of omniscience are largely failures of knowing all the alternatives, uncertainty about relevant exogenous events, and inability to calculate consequences."

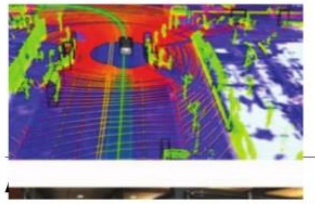




Definition of computational rationality

Gershman, Horvitz, & Tenembaum (2015) *Science*

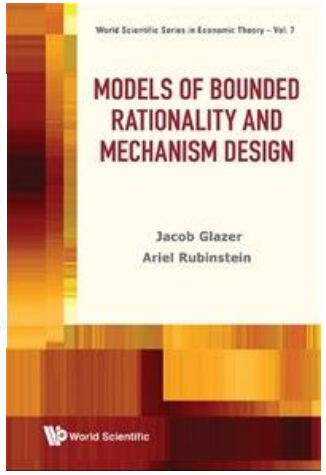
"Computing with representations, algorithms, and architectures designed to approximate decisions with the highest expected utility, while taking into account the costs of computation."



Models build on "inferential processes for perceiving, predicting, and reasoning under uncertainty"

Computational Rationality I – Antti Oulasvirta March 12, 2018

44



Neuroanatomical Substrates for Risk and Ambiguity Behavior

Psychologists joined by neuroscientists to reveal the neural mechanisms underlying uncertainty Behavior and to examine the neural bases of several rudimentary mental processes that contribute to uncertainty-taking/uncertainty-seeking behavior.

Individual Decision making

- **Risky** (relatively rare and found most in gambling)
  (known probability)
- **Ambiguity**
  (unknown/unknowable probability)
  - **Total ambiguity**
    (ambiguity aversion)
  - **Partial ambiguity**
    ambiguity + information
    - complete resolving ?
    - incomplete resolving ?

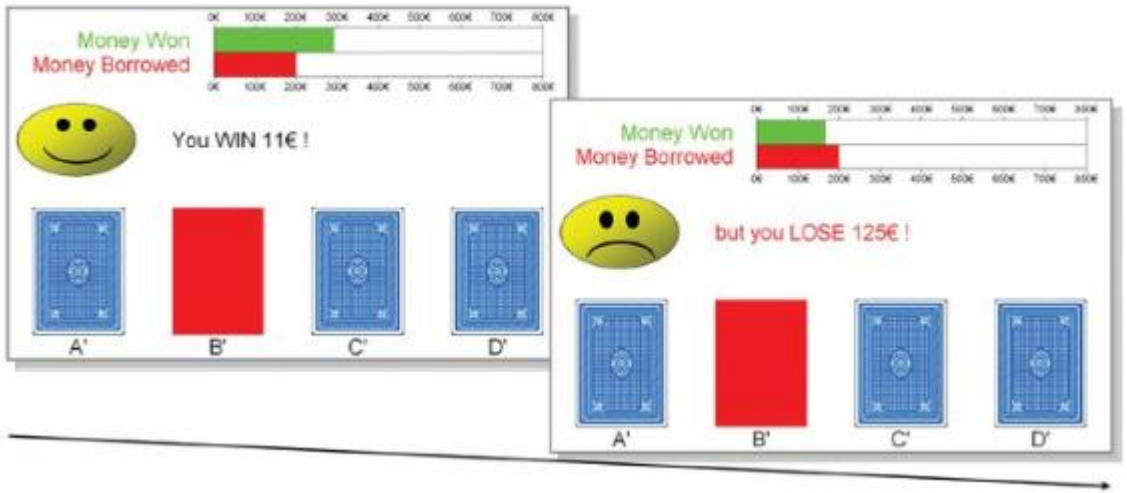
Subjective probability: Individuals vary substantially in how they perceive outcome probabilities

Risk attitude: how individuals tradeoff outcome magnitude against its Probability

Ambiguity attitude: how individuals treat ambiguity around outcome probabilities

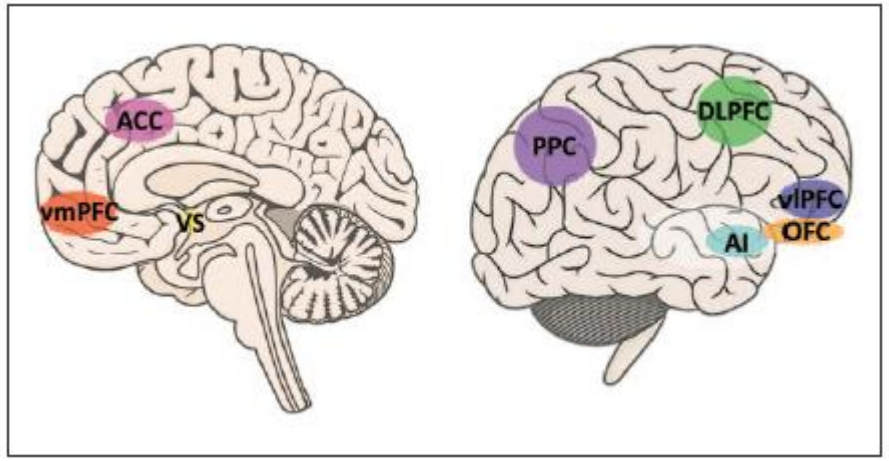
A widely used example of such task is the Iowa Gambling Task (IGT; Bechara and others 1997), in which participants learned to associate stimuli with uncertain rewards or punishments and to select the stimuli that lead to better outcomes.

1. participants are presented with four decks of cards, and on each trial draw a card from a deck of their choice.
2. Each card is associated with either a gain or a loss. In two of the decks, most of the cards lead to large gains, but every now and then a card leads to an even larger loss, resulting in an overall loss in the long run. In the other two decks cards lead to lower gains, but even lower losses, resulting in a net gain.
3. At the beginning of the task, participants have no information about outcome probabilities (complete *ambiguity).* With time, ambiguity is reduced and healthy participants learn to limit their card choices to the “good” decks



Bechara and others (1997)

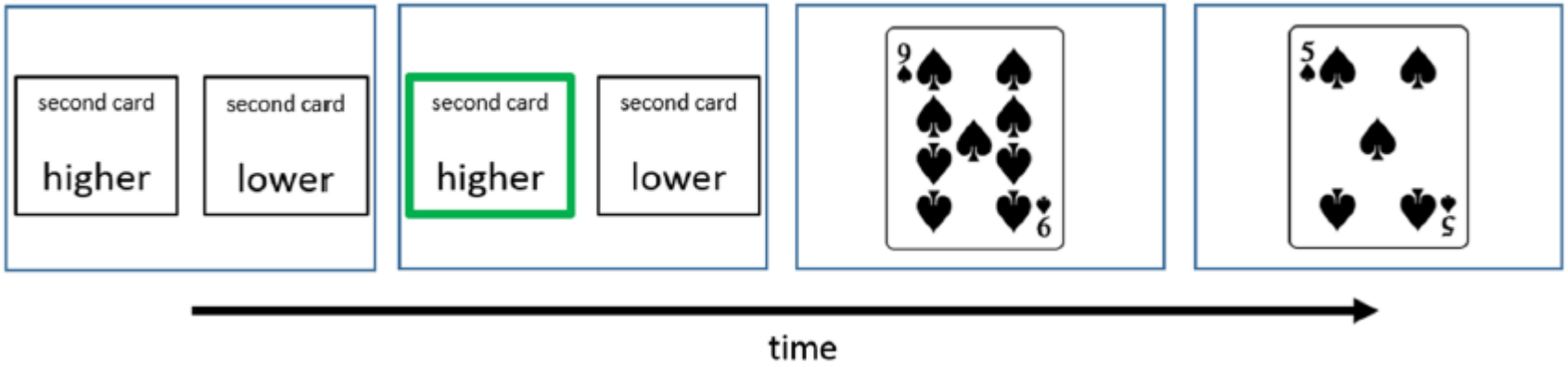
- Patients with brain lesion in ventromedial prefrontal cortex (vmPFC) are impaired on this task, supporting a role for vmPFC in risk-taking behavior and decision making in general.
- The complexity of the task, however, makes it difficult to delineate the specific role of vmPFC or the specific impairment in vmPFC-lesioned patients.
- In particular, learning from feedback is an important feature of the IGT, and reduced earnings on the task may result from a general learning impairment. Alternatively, deficient performance on the IGT may be due to overestimation of the positive value of potential gains, or underestimation of the negative value of potential losses. Therefore, while the IGT has been highly valuable in highlighting one of the central brain structures involved in decision making, many subsequent neuroimaging studies have opted for simpler designs, to allow for delineation of the neural bases of the various underlying cognitive processes.



AI = anterior insula; DLPFC = dorsolateral prefrontal cortex; OFC = orbitofrontal cortex; PPC = posterior parietal cortex; vlPFC = ventrolateral prefrontal cortex; vmPFC = ventromedial prefrontal cortex; VS = ventral striatum.
ACC = anterior cingulate cortex;

Betting on future events to eliminate the learning aspect (Preuschoff and others (2006))

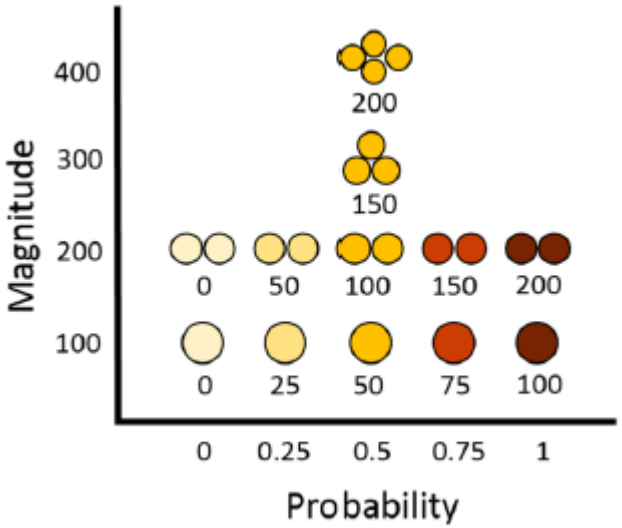
1. Participants bet on whether the second of two consecutively presented cards would be of a higher or lower number
2. Cards were withdrawn from a deck of 10 cards, numbered 1 to 10, with no repetition.
3. In this way, participants experienced varying levels of uncertainty during the anticipatory period between the presentation of the first and second cards. (For example, if the number on the first card was 1 or 10, participants could predict with certainty that the second card would bear a higher or lower number respectively. Conversely, uncertainty was maximal if the first card was numbered 5 or 6)

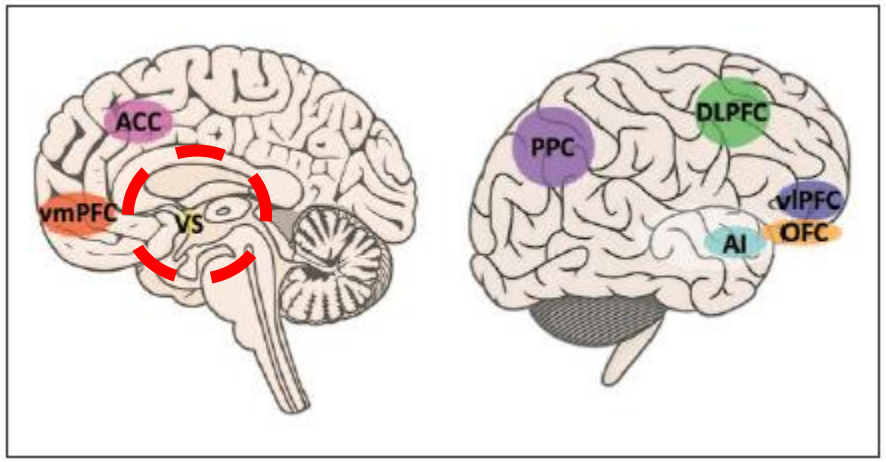


Increased activation to increased risk was observed in bilateral ventral striatum (Preuschoff and others 2006)

Conditioned Stimuli (Tobler and others (2007))

1. Presentation of stimuli that were previously associated with outcomes of particular magnitudes and probabilities
2. Unique stimuli were associated with a particular reward and a particular probability in an initial training session.
3. Following training, single stimuli, with fully established associations, were presented in the main experiment.

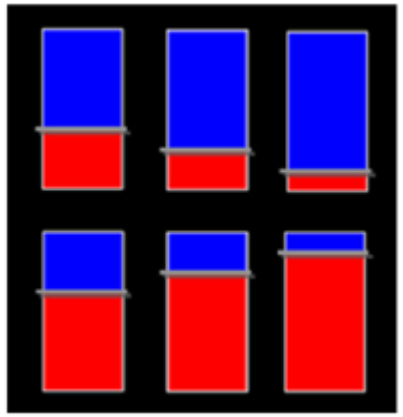




Activity scales positively with probability, compatible with the role of these areas in the encoding of *subjective value* or the desirability of anticipated outcomes.

Using “revealed preference” (Samuelson 1948).

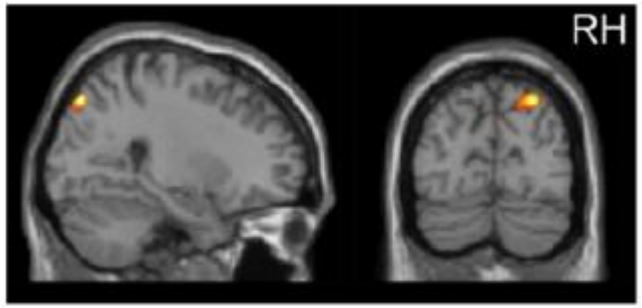
1. Participants choose between options that vary on their potential outcomes, as well as on the probability for obtaining these outcomes, and thus need to tradeoff reward and probability
2. Based on these observed choices, the researchers can estimate individual risk attitudes (for example, the choice between receiving $5 for sure and playing a lottery that offers 50% chance of winning $10 (but also 50% of winning nothing). Both options are of the same expected value, but the lottery is risky. An individual who is not affected by risk (risk neutral) will be indifferent between these options. Conversely, a risk-averse individual would prefer the sure $5, whereas a risk-seeking individual would opt for the lottery)
3. To simplify the design and the interpretation of the neural results, some of these studies keep one of the options constant across trials, such that any change in neural activation from trial to trial can be directly related to changes in only one option

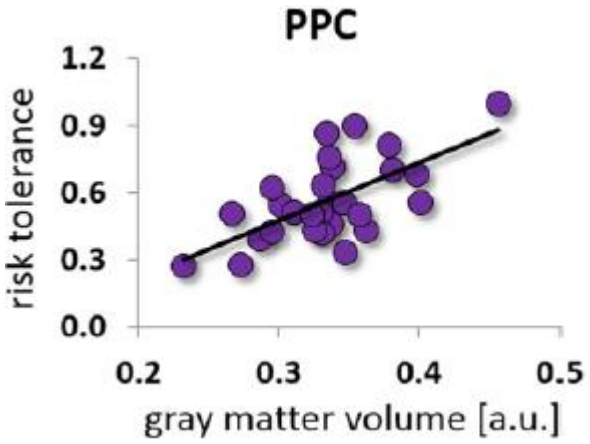


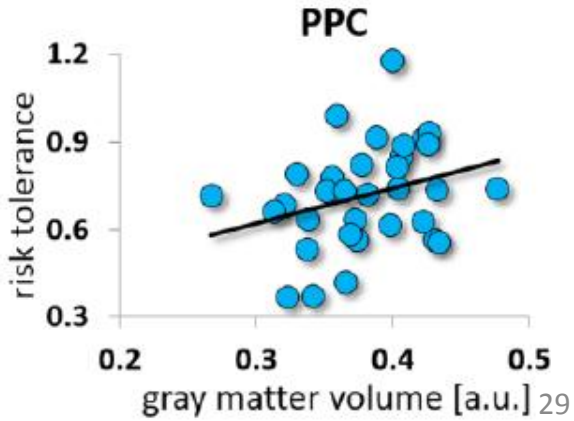
Gilaie-Dotan and others (2014)

Two groups of participants made a series of choices between risky options, and went through anatomical MRI scans.

1. The first group was tested in New York and provided data for a whole-brain exploratory analysis.
2. These attitudes were then used in a whole-brain VBM analysis, which revealed a single region, within right PPC, whose volume was significantly correlated with risk attitudes.
3. Individuals with more gray-matter volume in this region were more tolerant of risk (or less risk averse)
4. A similar result was obtained when risk attitudes were estimated simply based on the proportion of trials in which participants chose the risky option, demonstrating that the results did not depend on the specific assumptions used to calculate risk attitudes.
5. Following the exploratory analysis, data from a second, independent group of participants, scanned in Philadelphia, was used for a confirmatory analysis.





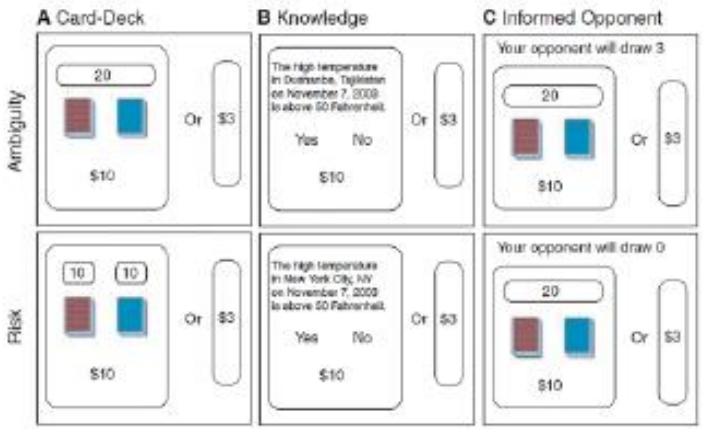


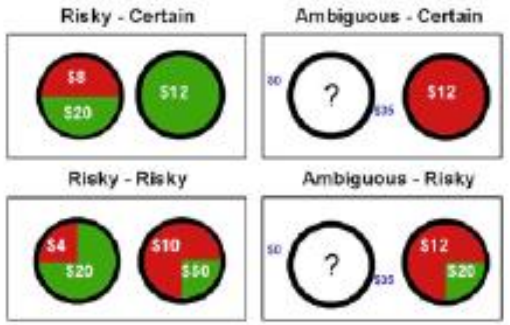
Ambiguous options are created by withholding some of the information about outcome probability, either by providing partial information or by physically occluding a graphic stimulus that conveys outcome probability.

Some of these studies compared complete ambiguity (i.e., no information about outcome probability) to no ambiguity (i.e., full information about outcome probability ; Hsu and others 2005; Huettel and others 2006).

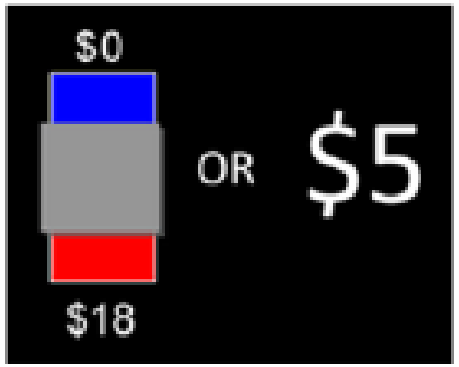
There is evidence for increased processing of ambiguity compared to risk in lateral OFC, which is correlated with the level of ambiguity aversion across participants (Hsu and others 2005).

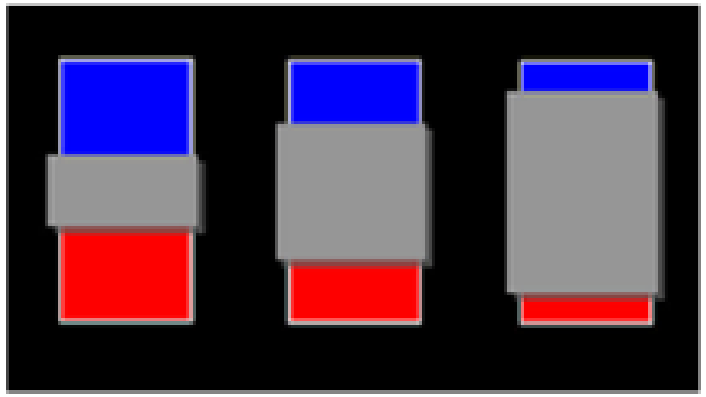
Taken together with the involvement of lateral OFC in encoding the level of risk (Huettel and others 2005; Tobler and others 2007), these findings suggest a general role for lateral OFC in uncertainty processing.
Activity in the neighboring region of vlPFC was associated with ambiguity preference across participants (Bach and others 2011; Huettel and others 2006)





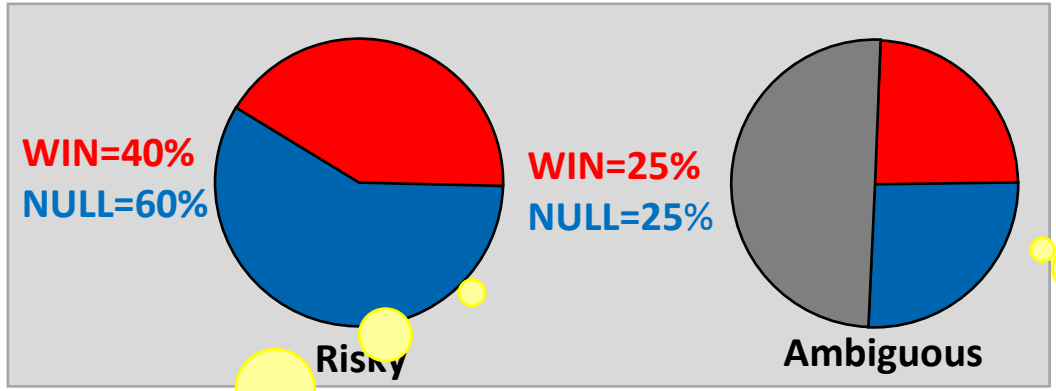
More recent studies included conditions of partial ambiguity by parametrically modulating the information provided in each trial.
Compatible with the effect of ambiguity on value, there is evidence for decreased activation in response to increasing ambiguity in the value-related vmPFC (Pushkarskaya and others 2015a).
Activity in the vmPFC, as well as in the striatum, is correlated with subjective value, which takes into account individual ambiguity (as well as risk) attitudes (Levy and others 2010).

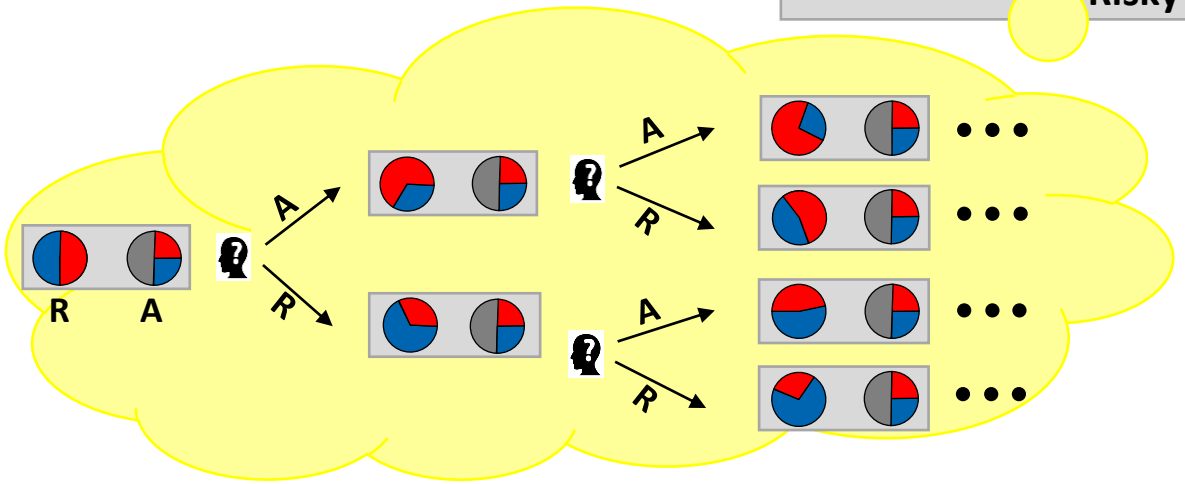


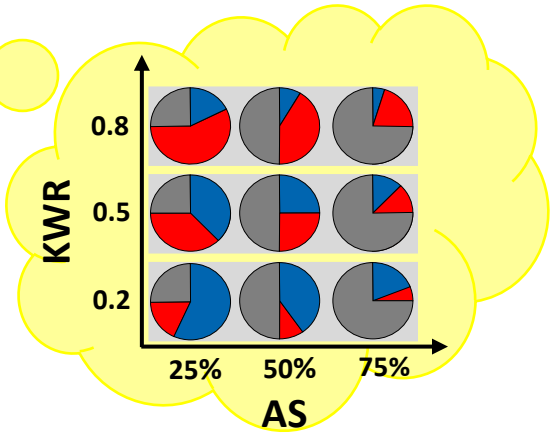


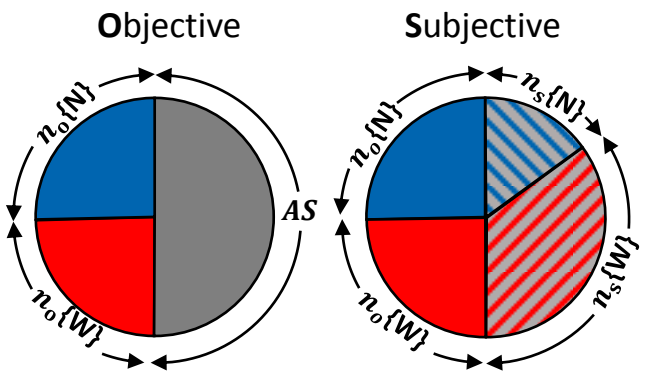
? How subjective belief about ambiguity may be constructed from positive vs. negative partial information

77 subjects (mean age=27.4, SD=4.3, 36 Females)



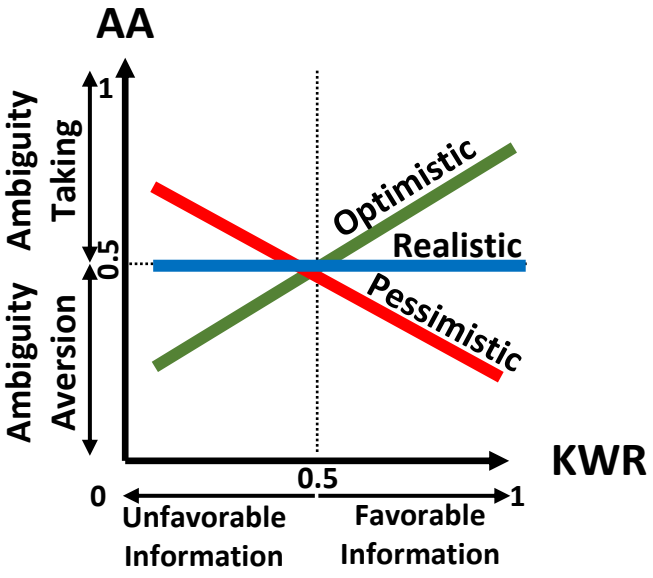






Prediction:

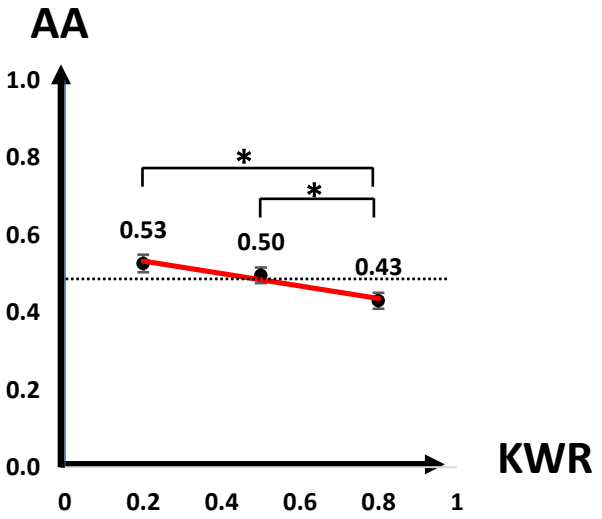
The ambiguity attitude would be greater in positive valence than the negative valence of information (Based on optimism bias)



$$AA = \frac{n_s\{W\}}{n_s\{W\} + n_s\{N\}}$$

Result:

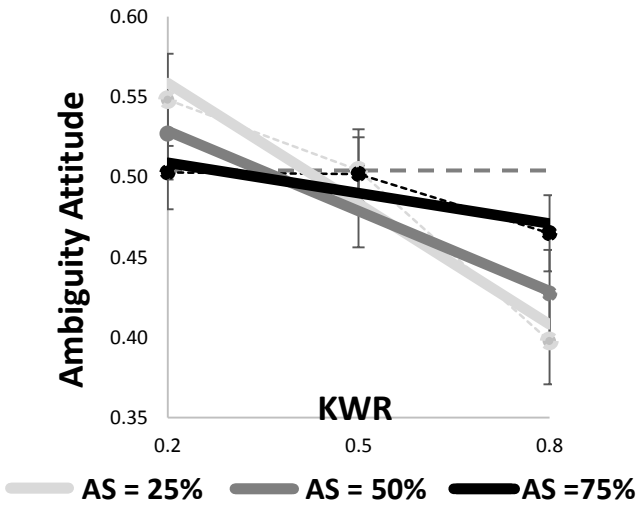
- **Main effect of KWR [F(2,679) = 12.18, p = 6.3e-6]**
- **No main effect of ambiguity size [F[2,679] = 0.14, p = 0.87]**
- **No interaction between the independent variables**

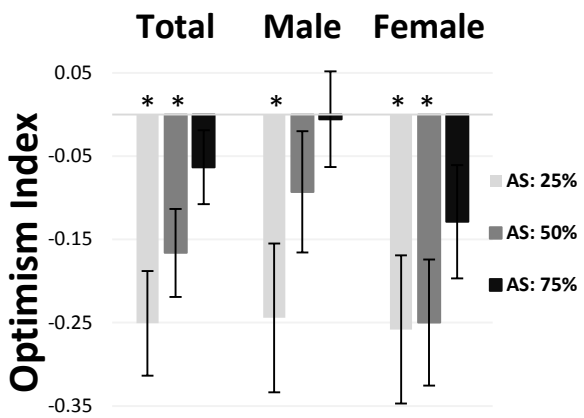


People don't trust the evidence to fill out the missing when deal with ambiguity

Result:

- When the ambiguity size was large, people tended to be more realistic
- When the ambiguity size was tractable, people tended to be pessimist





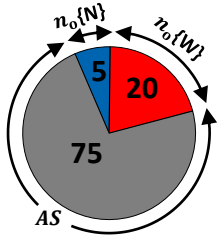
$$\boldsymbol{OI}\ (Optimism\ Index) = \frac{\Delta\ AA}{\Delta\ KWR}$$

- No main effect of gender on OIs (F[1,225] = 2.47 , p = 0.12)
- Main effect of AS on OIs (F[2,225] = 2.92, p = 0.05)
- No difference between OI in AS= 75% with zero (one sample t-test with zero; AS = 75%; t[76] = -1.42 , p = 0.16)
- The OIs in small and medium ambiguity size conditions were significantly less than zero (one sample t-test with zero; AS= 50%: t[76] =-3.14, p =0.0024; AS= 25%: t[76] = -3.99, p = 1.00E-04)

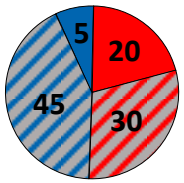
?How subjective belief is updated with the evidence about the structure of ignorance

Predictions from a number of possible alternative strategies

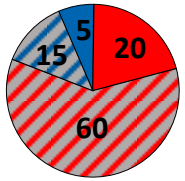
Objective



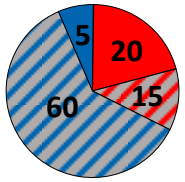
Variance Maximization



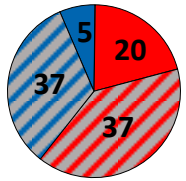
Direct Extrapolation



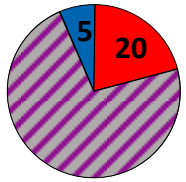
Reverse Extrapolation

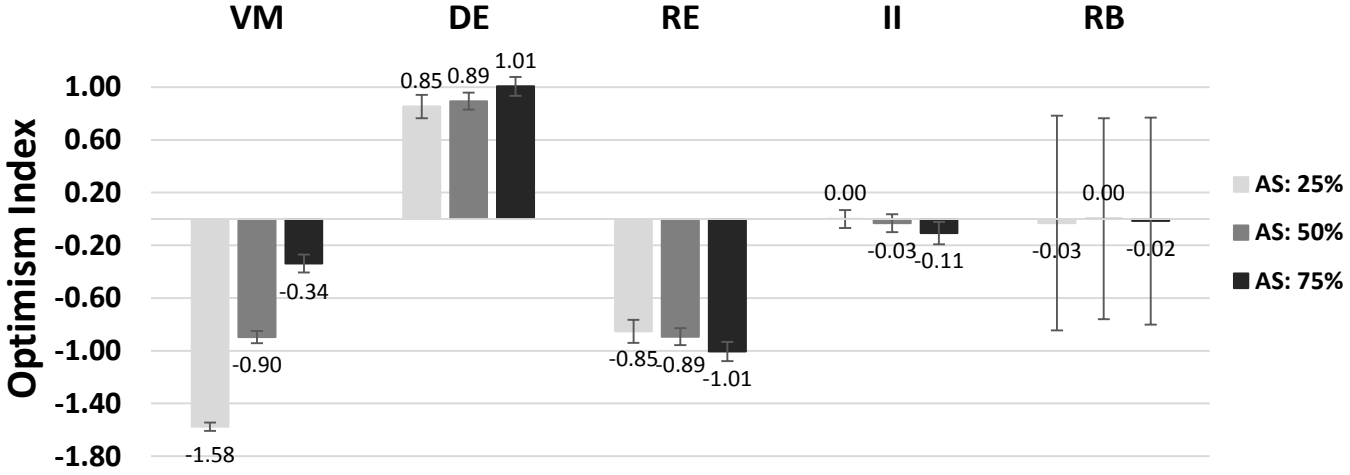


Ignore the Information

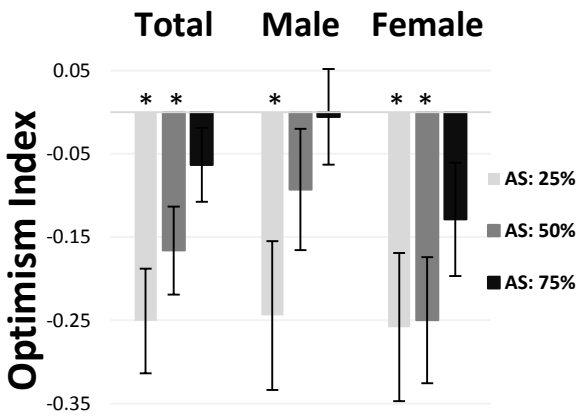


Random Behavior





?



Summary and Open Questions

While there is some evidence for consistent risk attitudes across rewards domains, specifically food and money (Levy and Glimcher 2011), there is also behavioral evidence for domain-specific uncertainty attitudes (Weber and others 2002), and substantial evidence for different risk and ambiguity attitudes for gains and losses (Tymula and others 2013).

Whether the same neural mechanisms support vastly different risk-taking behaviors, such as financial investments and medical decisions, remains to be seen.

Another interesting question is how much of this neural architecture is hardwired, and how much may be shaped by experience.

As decreased or increased risk-taking behavior is closely linked to psychopathology and substance abuse, understanding the relevant neural circuitry, its variations in pathological conditions, and how it may be modified by behavioral and pharmacological interventions is of high public health value.